

The use of this type of discretizing of molecular velocities has much wider application than that of representing a Maxwellian distribution function. Using this discrete ordinate method in rarefied flow problems transforms the Boltzmann equation with the Bhatnagar-Gross-Krook model from an integro-differential equation to a system of differential equations and simplifies calculations considerably. This method has proven successful in cases where departures from equilibrium are small [in which the discretization under discussion here is very useful since the g function of Eq. (1) is a perturbation about the Maxwellian] and also for those where large departures from equilibrium occur⁷⁻¹⁰ (in which this particular discretization is totally inadequate and different quadratures must be used).

There is one difference in the treatment of two- and three-dimensional distributions between Refs. 1 and 5. The authors of Ref. 1 are interested primarily in Maxwellian distributions and for higher dimensions use an energy variable rather than a velocity variable. However, for application of the discretization to nonequilibrium rarefied flow problems, particularly where a solid boundary is present, this cannot be done since one must distinguish between particles approaching a surface and those being emitted from a surface. This distinction is necessary in order to satisfy the boundary conditions for the problem. Thus, in gas-dynamic problems the two- and three-dimensional discretization takes the form of a multiple usage of the one-dimensional results, i.e.,

$$\int_0^\infty \iint g(v_x, v_y, v_z) e^{-(v_x^2 + v_y^2 + v_z^2)} d^3v = \sum_{i=1}^L \sum_{j=1}^M \sum_{k=1}^N H_i H_j H_k g(\alpha_i, \alpha_j, \alpha_k) \quad (3)$$

References

¹ Laframboise, J. G. and Stauffer, A. D., "Optimum Discrete Approximation of the Maxwell Distribution," *AIAA Journal*, Vol. 7, No. 3, March 1969, pp. 520-523.

² Huang, A. B. and Giddens, D. P., "The Discrete Ordinate Method for the Linearized Boundary Value Problems in Kinetic Theory of Gases," *Proceedings of 5th International Symposium on Rarefied Gas Dynamics*, edited by C. L. Brundin, Academic Press, New York, 1967, pp. 481-504.

³ Huang, A. B. and Giddens, D. P., "Rayleigh's Problem at Low Mach Numbers Based on Kinetic Theory," *AIAA Journal*, Vol. 5, No. 7, July 1967, pp. 1354-1356.

⁴ Huang, A. B., Giddens, D. P., and Bagnal, C. W., "Rarefied Gas Flow Between Parallel Plates Based on the Discrete Ordinate Method," *The Physics of Fluids*, Vol. 10, No. 3, March 1967, pp. 498-502.

⁵ Huang, A. B. and Giddens, D. P., "A New Table for a Modified (Half-Range) Gauss-Hermite Quadrature with an Evaluation of the Integral $\int_0^\infty e^{-u^2 - x/u} du$," *Journal of Mathematics and Physics*, Vol. 47, No. 2, June 1968, pp. 213-218.

⁶ Giddens, D. P., "Study of Rarefied Gas Flows by the Discrete Ordinate Method," Ph.D. thesis, 1966, Georgia Institute of Technology.

⁷ Huang, A. B., "A General Discrete Ordinate Method for the Dynamics of Rarefied Gases," Rept. No. 4, 1967, Aerospace Engineering Rarefied Gas Dynamics and Plasma Lab., Georgia Institute of Technology.

⁸ Huang, A. B., "Kinetic Theory of the Rarefied Supersonic Flow over a Finite Plate," *Proceedings of 6th International Symposium on Rarefied Gas Dynamics*, edited by L. Trilling and H. Y. Wachman, Academic Press, New York, 1969, pp. 529-544.

⁹ Huang, A. B. and Hartley, D. L., "Kinetic Theory of the Sharp Leading Edge Problem in Supersonic Flow," *The Physics of Fluids*, Vol. 12, No. 1, Jan. 1969, pp. 96-108.

¹⁰ Huang, A. B. and Hwang, P. F., "The Supersonic Leading Edge Problem According to the Ellipsoidal Model," AIAA Paper 69-652, San Francisco, 1969.

Reply by Authors to D. P. Giddens and A. B. Huang

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IT is unfortunate that we had not noticed the results of Huang and Giddens¹ prior to the publication of our results.² However, we feel that we have made substantial contributions beyond those of Ref. 1. We have published numbers for the two- and three-dimensional cases which are not available in Ref. 1. Although these numbers may not be useful in the application mentioned in the comment, our treatment of this problem was in fact motivated by an application of them (Ref. 2, Sec. III).

We have treated the one-dimensional case in which there is an odd number of levels, as well as the two- and three-dimensional cases in which negative moments must be duplicated. Finally, we have published numbers for the one-dimensional case which differ substantially from those of Ref. 1 and which we believe to be more accurate. To support this assertion, we have calculated

$$\sum_{j=1}^8 H_j \alpha_j^k \quad k = (0, 1, \dots, 15)$$

using the numbers in Ref. 1 for $n = 8$. According to the derivation of the roots and weights, these sums should give exact results for the corresponding moments. Yet errors ranging from 0.004% to 3.1% occur in the range $k = 8, 9, \dots, 15$. In contrast, using the numbers from Ref. 2 gives results which are in error by less than one part in 10^6 . In fact these numbers, prior to rounding off, recover these moments to one part in 10^{12} or better. We conclude that the numbers in Ref. 2 should be used in any calculations performed in preference to those of Ref. 1.

References

¹ Huang, A. B. and Giddens, D. P., "A New Table for a Modified (Half-Range) Gauss-Hermite Quadrature with an Evaluation of the Integral $\int_0^\infty e^{-u^2 - (x/u)} du$," *Journal of Mathematics and Physics*, Vol. 47, No. 2, June 1968, pp. 213-218.

² Laframboise, J. G. and Stauffer, A. D., "Optimum Discrete Approximation of the Maxwell Distribution," *AIAA Journal*, Vol. 7, No. 3, March 1969, pp. 520-523.

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Erratum: "Further Similar Laminar-Flow Solutions"

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REFERENCE 5 of this paper should read: Rogers, D. F., "Reverse Flow Solutions for Compressible Laminar Boundary Layer Equations," *The Physics of Fluids*, Vol. 12, No. 3, March 1969, pp. 517-523.